

# Gain

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**Chapter 2**  
*Fundamental Parameters of Antennas*

$$P_{rad} = e_c e_d P_{in}$$

$$P_{rad} = e_{cd} P_{in}$$

$$\text{Gain} = G = 4\pi \frac{\text{Radiation intensity}}{\text{Total input (accepted) power}}$$

$$G = 4\pi \frac{U(\theta, \phi)}{P_{in}} \quad (2-46)$$

$$P_{rad} = e_{cd} P_{in} \Rightarrow P_{in} = \frac{P_{rad}}{e_{cd}} \quad (2-47)$$

$$G = 4\pi \frac{U(\theta, \phi)}{P_{rad}/e_{cd}} = e_{cd} \underbrace{\left[ 4\pi \frac{U(\theta, \phi)}{P_{rad}} \right]}_D \quad (2-48)$$

$$G = e_{cd} D$$

$$G_o = e_{cd} D_o$$

(2-49a)

$e_{cd} = e_c e_d = \text{Radiation efficiency}$

$e_c = \text{Conduction efficiency}$

$e_d = \text{Dielectric efficiency}$

# Absolute Gain $G_{abs}$

$$\begin{aligned} G_{abs}(\theta, \phi) &= e_o D(\theta, \phi) = e_r e_{cd} D(\theta, \phi) \\ &= (1 - |\Gamma_{in}|^2) e_{cd} D(\theta, \phi) \end{aligned} \quad (2-49b)$$

$e_o$  = antenna total efficiency

$e_r = (1 - |\Gamma_{in}|^2)$  = Reflection efficiency

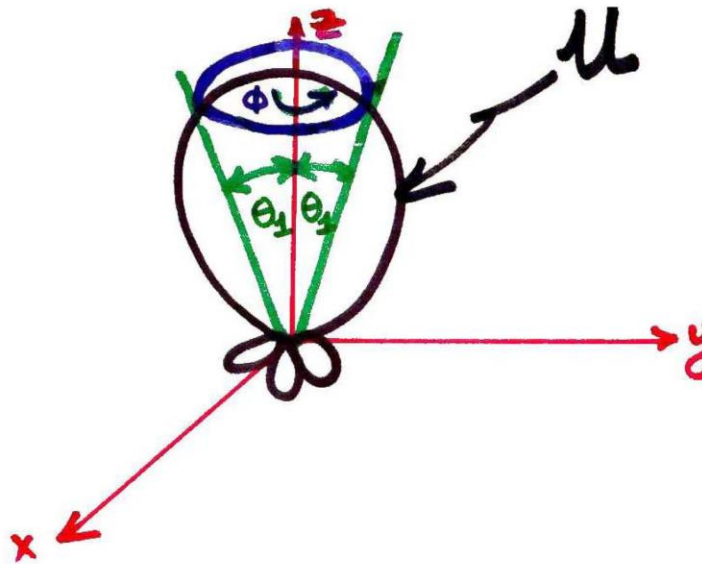
$e_{cd} = e_c e_d$  = Radiation efficiency

$e_r$  = Conduction efficiency

$e_d$  = Dielectric efficiency

# Beam Efficiency

$$BE = \frac{\int_0^{2\pi} \int_0^{\theta_1} U(\theta, \phi) \sin \theta d\theta d\phi}{\int_0^{2\pi} \int_0^{\pi} U(\theta, \phi) \sin \theta d\theta d\phi} \quad (2-54)$$

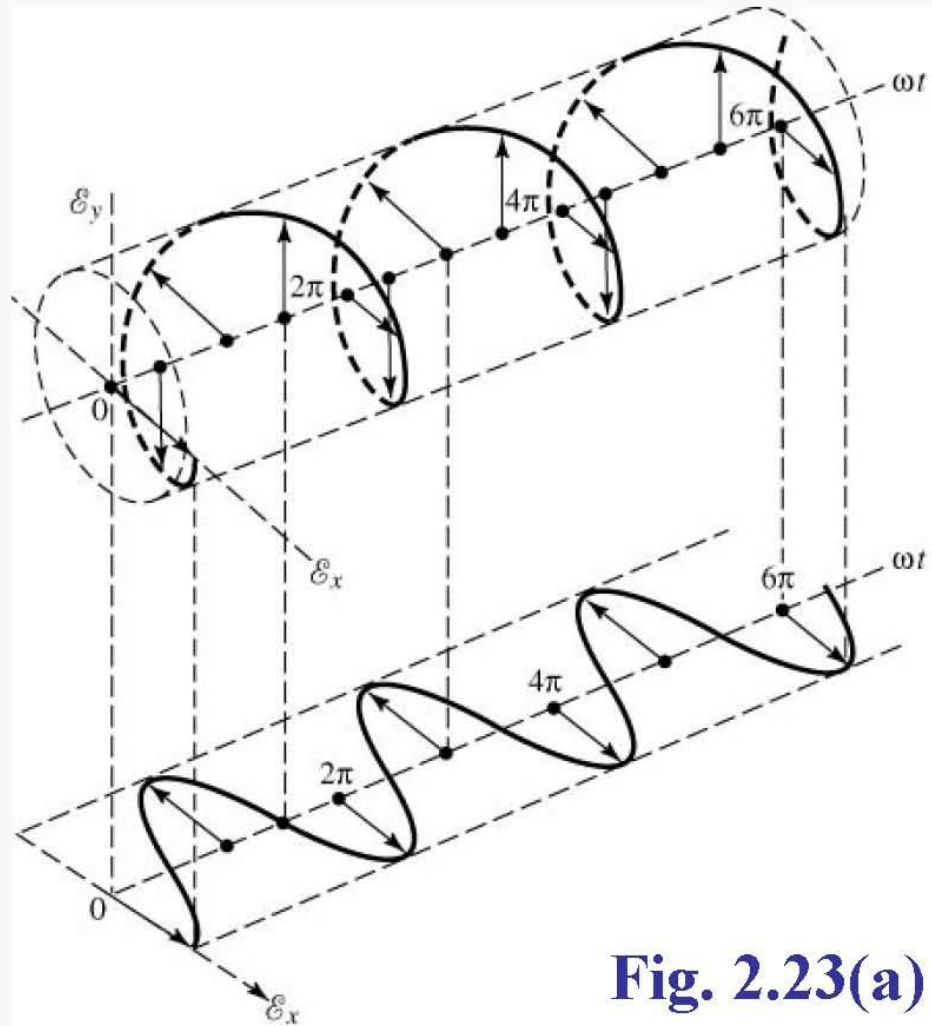


# Polarization

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# Rotation of Wave



**Fig. 2.23(a)**

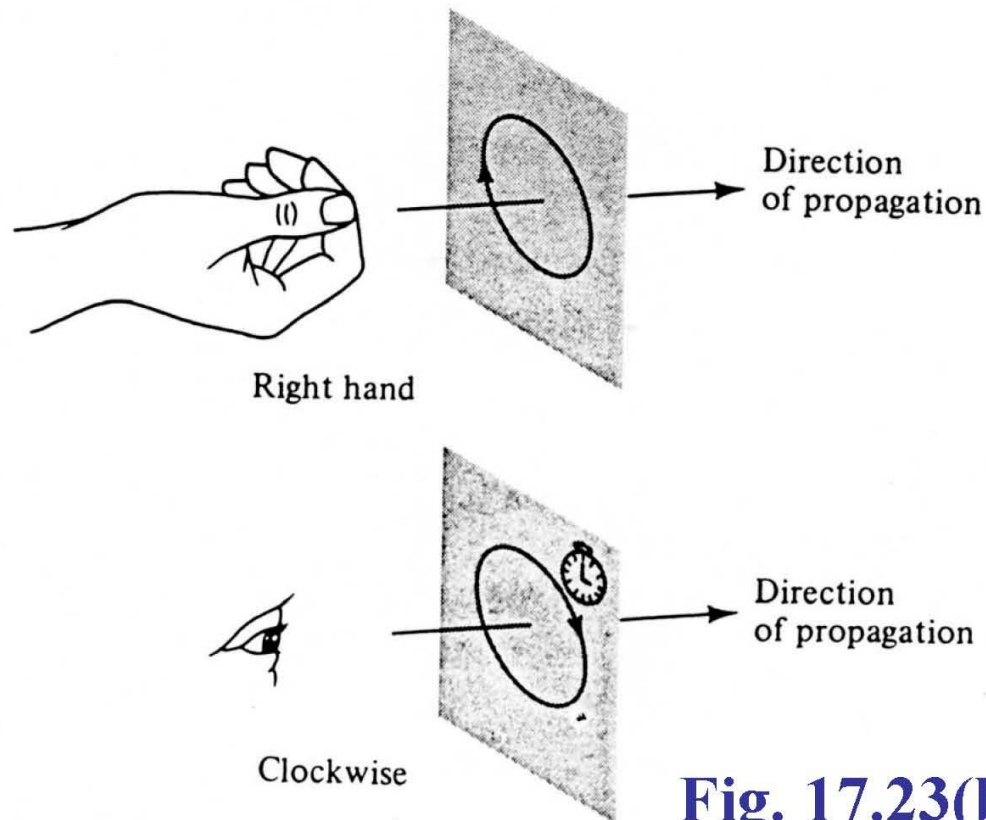
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# Polarization Ellipse & Sense of Rotation for Antenna Coordinate System

## Sense Of Rotation



**Fig. 17.23(b)**

# Polarization

A. Linear

B. Circular

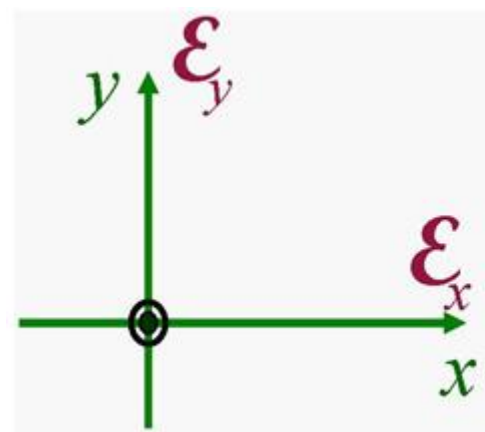
1. CW (RH)

2. CCW (LH)

C. Elliptical (**Axial Ratio**)

1. CW (RH)

2. CCW (LH)



$$\underline{E} = \left[ \hat{a}_x E_{x_0} + \hat{a}_y E_{y_0} e^{j\Delta\phi} \right] e^{+jkz}$$

### I. Linear

A.  $E_{x_0} \neq 0, E_{y_0} = 0$

B.  $E_{x_0} = 0, E_{y_0} \neq 0$

C.  $E_{x_0} \neq 0, E_{y_0} \neq 0$

$$\Delta\phi = \pm n\pi, \quad n = 0, 1, 2, \dots$$

### II. Circular

$$E_{x_0} = E_{y_0} \quad (2-59)$$

$$\Delta\phi = \pm \left( \frac{1}{2} + n \right) \pi, \quad n = 0, 1, 2, \dots \quad (2-60, -61)$$

+ : clockwise (RH)

- : counterclockwise (LH)

### III. Elliptical

A.  $E_{x_0} \neq E_{y_0}, \Delta\phi \neq \pm n\pi, \quad n = 0, 1, 2, \dots$

B.  $E_{x_0} = E_{y_0}, \Delta\phi \neq \pm \left( \frac{1}{2} + n \right) \pi, \quad n = 0, 1, 2, \dots$   
(2-62a,b)

$$\text{AR} = \frac{\text{major axis}}{\text{minor axis}} = \frac{OA}{OB} \quad 1 \leq \text{AR} \leq \infty \quad (2-65)$$

# Polarization Loss Factor (PLF)

$$PLF = |\hat{\rho}_w \cdot \hat{\rho}_a|^2 = |\cos \psi_p|^2$$

$$0 \leq PLF \leq 1$$

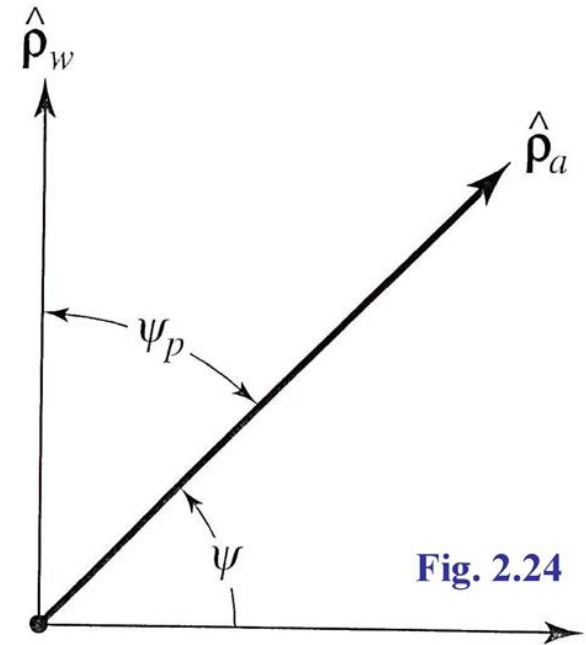
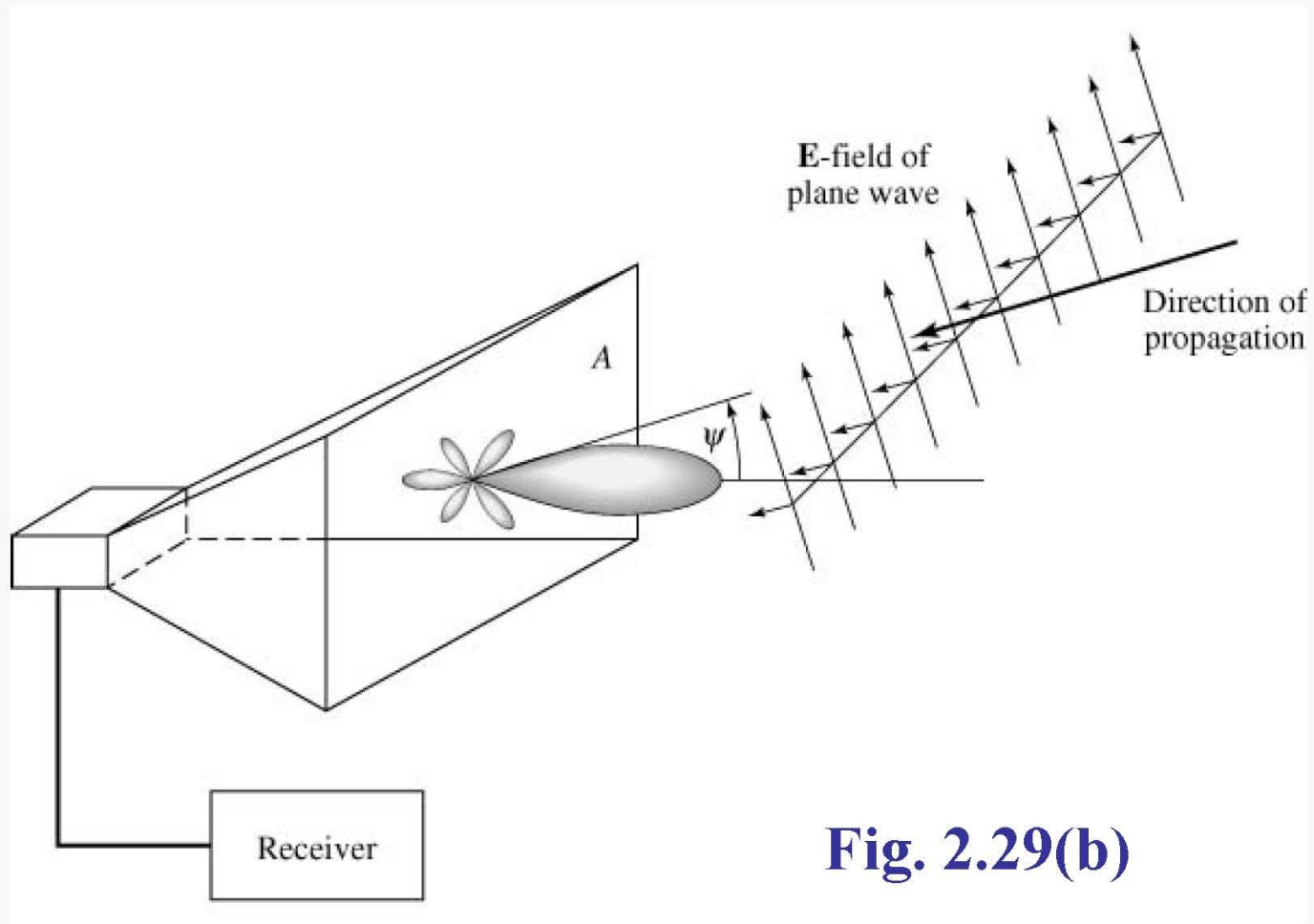


Fig. 2.24

# Effective Aperture (Area) $A_e$

# Aperture Antenna in Receiving Mode



**Fig. 2.29(b)**

$$A_e = \frac{\lambda^2}{4\pi} \underbrace{e_o D(\theta, \phi)}_{G_{abs}(\theta, \phi)} |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

$$A_{em} = \frac{\lambda^2}{4\pi} \underbrace{e_o D_0}_{G_{0abs}} |\hat{\rho}_w \cdot \hat{\rho}_a|^2$$

$$A_{em} = \frac{\lambda^2}{4\pi} D_0 \underbrace{e_{cd} (1 - |\Gamma|^2)}_{e_o} \underbrace{|\hat{\rho}_w \cdot \hat{\rho}_a|^2}_{e_p = PLF} \quad (2-112)$$



## Very Maximum (no losses) $A_{em}$

$$A_{em} = \frac{\lambda^2}{4\pi} D_o \quad (2-110)$$



# Aperture Efficiency ( $\epsilon_{ap}$ )

$$\epsilon_{ap} = \frac{A_{em}}{A_p} \quad (2-100)$$

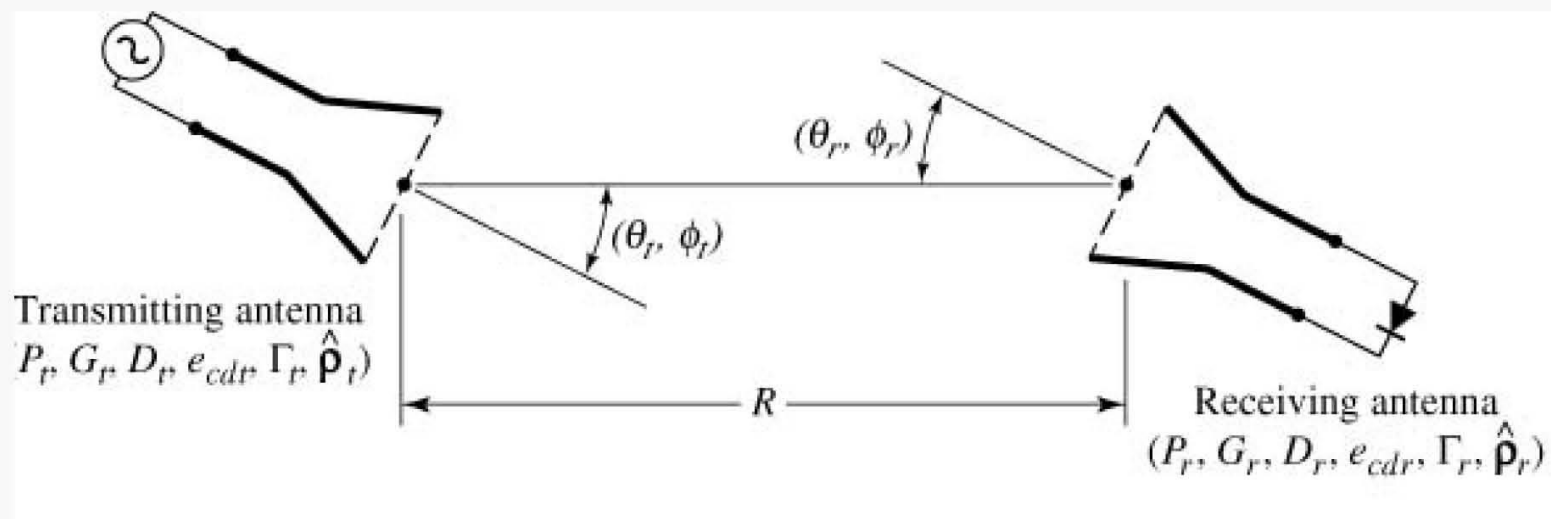
For aperture Antennas:

$$A_{em} \leq A_p \Rightarrow \epsilon_{ap} \leq 1$$

# Friis Transmission Equation

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**Fig. 2.31**

$$\frac{P_r}{P_t} = e_t e_r \frac{\lambda^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r)}{(4\pi R)^2} \quad (2-217)$$

$$\frac{P_r}{P_t} = e_{cdt} e_{cdr} \left(1 - |\Gamma_t|^2\right) \left(1 - |\Gamma_r|^2\right) \left(\frac{\lambda}{4\pi R}\right)^2 D_t D_r$$

$$\frac{P_r}{P_t} = e_{cdt} e_{cdr} \left(1 - |\Gamma_t|^2\right) \left(1 - |\Gamma_r|^2\right) \left(\frac{\lambda}{4\pi R}\right)^2 D_t D_r |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

$$\frac{P_r}{P_t} = \left(1 - |\Gamma_t|^2\right) \left(1 - |\Gamma_r|^2\right) \left(\frac{\lambda}{4\pi R}\right)^2 G_t G_r |\hat{\rho}_t \cdot \hat{\rho}_r|^2$$

## Assuming No Losses:

$$\frac{P_r}{P_t} = D_{ot} D_{or} \left( \frac{\lambda}{4\pi R} \right)^2$$

## More General: Includes Losses:

$$\frac{P_r}{P_t} = G_{ot} G_{or} \underbrace{\left( \frac{\lambda}{4\pi R} \right)^2}_{\text{Free space loss factor}} \left| \hat{\rho}_t \cdot \hat{\rho}_r \right|^2 \quad (2-119)$$